Colour Space Conversions

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Contents

1 Introduction

This document consists of equations to transform between many different colour spaces which are common in the fields of computer generated and computer displayed imagery. It is strongly recommended that readers of this document also read Charles Poynton's excellent FAQ's on color and gamma. They can be found (in a wide range of formats) at:

`http://www.poynton.com/Poynton-color.html`

The latest version of this document can be found at:

`http://www.poynton.com/PDFs/coloureq.pdf`

At 2001-07-20, neither Adrian Ford nor Alan Roberts is actively maintaining this document; it is being distributed and passively maintained by Charles Poynton (poynton@poynton.com).

Before using any information in this document you are advised to read section 15.

2 Some Colour Definitions and Explanations.

Colour is extremely subjective and personal. To try to attribute numbers to the brains reaction to visual stimuli is very difficult. The aim of colour spaces is to aid the process of describing colour, either between people or between machines or programs.

2.1 What is the correct way to describe colour?

Colour is the brains reaction to a specific visual stimulus. Although we can precisely describe colour by measuring its spectral power distribution (the intensity of the visible electro-magnetic radiation at many discrete wavelengths) this leads to a large degree of redundancy. The reason for this redundancy is that the eye's retina samples colour using only three broad bands, roughly corresponding to red, green and blue light. The signals from these colour sensitive cells (cones), together with those from the rods (sensitive to intensity only), are combined in the brain to give several different "sensations" of the colour. These sensations have been defined by the CIE (see section 5.1) and are quoted from Hunt's book "Measuring Colour":

- Brightness: the human sensation by which an area exhibits more or less light.

- Hue: the human sensation according to which an area appears to be similar to one, or to proportions of two, of the perceived colours red, yellow, green and blue.
- Colourfulness: the human sensation according to which an area appears to exhibit more or less of its hue.
- Lightness: the sensation of an area's brightness relative to a reference white in the scene.
- Chroma: the colourfulness of an area relative to the brightness of a reference white.
- Saturation: the colourfulness of an area relative to its brightness.

The tri-chromatic theory describes the way three separate lights, red, green and blue, can match any visible colour – based on the eye's use of three colour sensitive sensors. This is the basis on which photography and printing operate, using three different coloured dyes to reproduce colour in a scene. It is also the way that most computer colour spaces operate, using three parameters to define a colour.

2.2 What is a colour space?

A colour space is a method by which we can specify, create and visualise colour. As humans, we may define a colour by its attributes of brightness, hue and colourfulness. A computer may describe a colour using the amounts of red, green and blue phosphor emission required to match a colour. A printing press may produce a specific colour in terms of the reflectance and absorbance of cyan, magenta, yellow and black inks on the printing paper.

A colour is thus usually specified using three co-ordinates, or parameters. These parameters describe the position of the colour within the colour space being used. They do not tell us what the colour is, that depends on what colour space is being used.

An analogy to this is that I could tell you where I live by giving directions from the local garage, those directions only mean anything if you know the location of the garage before hand. If you don't know where the garage is the instructions are meaningless.

2.3 Why is there more than one colour space?

Different colour spaces are better for different applications, for example some equipment has limiting factors that dictate the size and type of colour space that can be used.

Some colour spaces are perceptually linear, i.e. a 10 unit change in stimulus will produce the same change in perception wherever it is applied. Many colour spaces, particularly in computer graphics, are not linear in this way.

Some colour spaces are intuitive to use, i.e. it is easy for the user to navigate within them and creating desired colours is relatively easy. Other spaces are confusing for the user with parameters with abstract relationships to the perceived colour.

Finally, some colour spaces are tied to a specific piece of equipment (i.e. are device dependent) while others are equally valid on whatever device they are used.

2.4 What's the difference between device dependent and device independent?

A device dependent colour space is a colour space where the colour produced depends both the parameters used and on the equipment used for display. For example try specifying the same RGB values on two different workstations, the colour produced will be visually different if viewed on side by side screens. Another test is to change the brightness and contrast settings (the offset and gain) on the CRT and see how a displayed colour varies. A device independent colour space is one where a set of parameters will produce the same colour on whatever equipment they are used.

One way to visualise this difference is to return to our directions problem. I can specify where I live uniquely by giving exact longitude and latitude parameters. Alternatively I can use a grid reference from a UK OS map. The OS grid reference is fine if you're in the UK (or if you can translate between the grid ref and longitude-latitude) the grid reference can be thought of as a device dependent specification. The longitude – latitude specification however is device independent since it has the same meaning to anyone from anywhere.

Some device dependent colour spaces are well characterised so that the user can translate between them and some other, device independent, colour space. Typically this characterisation takes the form of specifying the chromaticities (exact measured colour – see section 5.1) of the three primaries as well as the transfer functions for each channel (see section 3). Such colour spaces are known as device calibrated colour spaces and are a kind of half way house between dependent and independent colour spaces.

2.5 What is a colour gamut?

A colour gamut is the area enclosed by a colour space in three dimensions. It is usual to represent the gamut of a colour reproduction system graphically as the range of colours available in some device independent colour space. Often the gamut will be represented in only two dimensions, for example on a CIE u'v' chromaticity diagram (see section 5.1).

2.6 What is the CIE System?

The CIE has defined a system that classifies colour according to the HVS (the human visual system). Using this system we can specify any colour in terms of its CIE co-ordinates and hence be confident that a CIE defined colour will match another with the same CIE definition.

A brief a superficial description follows below. Fuller details are contained in Hunt's book, "Measuring Colour".

The CIE has measured the sensitivities of the three broad bands in the eye by matching spectral colours to specific mixtures of three coloured lights. The spectral power distribution (SPD) of a colour is cascaded with these sensitivity functions to produce three tri-stimulus values. These tri-stimulus values uniquely represent a colour, however since the illuminant and lighting and viewing geometry will affect the measurements these are all carefully defined. The three CIE tri-stimulus values are the building blocks from which many colour specifications are made.

2.7 What colour space should I use?

That depends on what you want to do; but here is a list of the pros and cons of some of the more common, computer related, colour spaces;

2.7.1 RGB (Red Green Blue)

This is an additive colour system based on tri-chromatic theory. Often found in systems that use a CRT to display images. RGB is easy to implement but non–linear with visual perception. It is device dependent and specification of colours is semi–intuitive. RGB is very common, being used in virtually every computer system as well as television, video etc.

2.7.2 CMY(K) (Cyan Magenta Yellow (Black))

This is a subtractive based colour space and is mainly used in printing and hard copy output. The fourth, black, component is included to improve both the density range and the available colour gamut (by removing the need for the CMY inks to produce a good neutral black it is possible to used inks that have better colour reproductive capabilities).

CMY(K) is fairly easy to implement but proper transfer from RGB to CMY(K) is very difficult (simple transforms are, to put it bluntly, simple). CMY(K) is device dependent, non–linear with visual perception and reasonably unintuitive.

2.7.3 HSL (Hue Saturation and Lightness)

This represents a wealth of similar colour spaces, alternative names include HSI (intensity), HSV (value), HCI (chroma / colourfulness), HVC, TSD (hue saturation and darkness) etc. Most of these colour spaces are linear transforms from RGB and are therefore device dependent and non–linear. Their advantage lies in the extremely intuitive manner of specifying colour. It is very easy to select a desired hue and to then modify it slightly by adjustment of its saturation and intensity.

The supposed separation of the luminance component from chrominance (colour) information is stated to have advantages in applications such as image processing. However the exact conversion of RGB to hue, saturation and lightness information depends entirely on the equipment characteristics. Failure to understand this may account for the sheer numbers of related but different transforms of RGB to HSL, each claimed to be better for specific applications than the others.

2.7.4 YIQ, YUV, YCbCr, YCC (Luminance - Chrominance)

These are the television transmission colour spaces, sometimes known as transmission primaries. YIQ and YUV are analogue spaces for NTSC and PAL systems respectively while YCbCr is a digital standard.

These colour spaces separate RGB into luminance and chrominance information and are useful in compression applications (both digital and analogue). These spaces are device dependent but are intended for use under strictly defined conditions within closed systems. They are also quite unintuitive, unless of course you are a TV engineer.

Kodak uses a derivative of YCC in its PhotoCD system, PhotoYCC.

2.7.5 CIE

There are two CIE based colour spaces, CIELuv and CIELab. They are nearly linear with visual perception, or at least as close as any colour space is expected to sensibly get. Since they are based on the CIE system of colour measurement, which is itself based on human vision, CIELab and CIELuv are device independent but suffer from being quite unintuitive despite the L parameter having a good correlation with perceived lightness.

To make them more user friendly, the CIE defined two analogous spaces - CIELhs or CIELhc where h stands for hue, s for saturation and c for chroma. In addition CIEluv has an associated two-dimensional chromaticity chart which is useful for showing additive colour mixtures, making CIELuv useful in applications using CRT displays.

CIELab has no associated two dimensional chromaticity diagram and no correlate of saturation. CIELhs can therefore not be defined.

3 Gamma and linearity.

Many image processing operations, and also colour space transforms that involve device independent colour spaces, must be performed in a linear luminance domain. By this we really mean that the relationship between image pixel values specified in software and the luminance of a specific area on the CRT display must be known. In most cases the CRT will have a nonlinear response. The luminance of a CRT is generally modelled using a power function with an exponent, gamma, somewhere between 2.2 (NTSC and SMPTE specifications) and 2.8 (as given by Hunt and Sproson). The common relationship is given below:

$$\text{Luminance} = \text{voltage}^{\gamma} \tag{1}$$

Where luminance and voltage are normalised between 0 and 1.

In order to display image information as linear luminance we need to modify the voltages sent to the CRT. This process stems from television systems where the camera and receiver had different transfer functions (which, unless corrected, would cause problems with tone reproduction). The modification applied is known as gamma correction and is given in equation 2.

$$\text{New Voltage} = \text{Old Voltage}^{\frac{1}{\gamma}} \tag{2}$$

(both voltages are normalised and gamma is the value of the exponent of the power function that most closely models the luminance-voltage relationship of the display being used.)

For a colour computer system we can replace the voltages by the image pixel values (assuming that your graphics card converts digital values to analogue voltages in accordance with the power relationship)as in equation 4.

$$\begin{aligned} R &= aR'^{\gamma} + b \\ G &= aG'^{\gamma} + b \end{aligned} \tag{3}$$

$$B = aB'^{\gamma} + b$$

where R', G', and B' are the normalised input Red, Green and Blue pixel values and R, G, and B are the normalised gamma corrected signals sent to the graphics card. The values of the constants a and b compensate for the overall system gain and system offset respectively (essentially gain is contrast and offset is intensity). For basic applications the value of a, b and gamma can be assumed to be consistent between colour channels, however for precise work they should be measured for each channel separately.

A more accurate description of the gamma relationship has recently been given in a paper by Berns et al while Charles Poynton's paper and his document GammaFAQ give a clear and concise description of the relationship and how it can be tackled across different computing platforms. The implementation of gamma correction for television standards is discussed more fully in section 10.

As a side note, gamma correction in 8-bit integer maths leads to substantial quantisation errors. Whenever possible perform gamma correction at the image acquisition stage as many scanners work with 10 or 12 bits per channel which will help to minimise distortion.

A final point is that the overall tone reproduction of an image will depend on the characteristics of acquisition, manipulation and display stages. In addition the preferred tone reproduction will vary according to viewing conditions. All these points should be considered when implementing gamma manipulation and correction in an application.

4 Grassman's Laws of additive colour mixture.

Any colour (source C) can be matched by a linear combination of three other colours (primaries e.g. RGB), provided that none of those three can be matched by a combination of the other two. This is fundamental to colorimetry and is Grassman's first law of colour mixture. So a colour C can be matched by Rc units of red, Gc units of green and Bc units of blue. The units are can be measured in any form that quantifies light power.

$$C = Rc(R) + Gc(G) + Bc(B) \tag{4}$$

A mixture of any two colours (sources C1 and C2) can be matched by linearly adding together the mixtures of any three other colours that individually match the two source colours. This is Grassman's second law of colour mixture. It can be extended to any number of source colours.

$$C3(C3) = C1(C1) + C2(C2) = [R1 + R2](R) + [G1 + G2](G) + [B1 + B2](B) \tag{5}$$

Colour matching persists at all luminances. This is Grassman's third law. It fails at very low light levels where rod cell vision (scoptopic) takes over from cone cell vision (photopic).

$$kC3(C3) = kC1(C1) + kC2(C2). \tag{6}$$

The symbols in square brackets are the names of the colours, and not numerical values. The equality sign should not be used to signify an identity, in colorimetry it means a colour matching, the colour on one side of the equality looks the same as the colour on the other side.

These laws govern all aspects of additive colour work, but they apply only signals in the "linear-light" domain. They can be extended into subtractive colour work.

5 Tristimuli, Chromaticity, and Colorimetric systems.

A colour can be described as a mixture of three other colours or "Tristimuli". Typically RGB for CRT based systems (TV, computer) or XYZ (fundamental measurements). The amounts of each stimulus define the colour. However, it is frequently useful to separate the colour definition into "luminance" and "chromaticity". Lower case is always used to signify chromaticity co-ordinates, upper case always signifies tristimulus values (or amounts of the primaries). Chromaticity co-ordinates can be plotted on a two-dimensional diagram that defines all the visible colours, luminance is normal to that diagram.

5.1 CIE XYZ (1931)

The CIE XYZ (1931) system is at the root of all colorimetry. It is defined such that all visible colours can be defined using only positive values, and, the Y value is luminance. Consequently, the colours of the XYZ primaries themselves are not visible. The chromaticity diagram is highly non-linear, in that a vector of unit magnitude representing the difference between two chromaticities is not uniformly visible. A colour defined in this system is referred to as Yxy. A third co-ordinate, z, can also be defined but is redundant since x+y+z=1 for all colours.

$$x = \frac{X}{(X+Y+Z)} \tag{7}$$

$$y = \frac{Y}{(X+Y+Z)} \tag{8}$$

5.2 CIE YUV (1960)

This is a linear transformation of Yxy, in an attempt to produce a chromaticity diagrams in which a vector of unit magnitude (difference between two points representing two colours) is equally visible at all colours. Y is unchanged from XYZ or Yxy. Difference non-uniformity is reduced considerably, but not enough. A third co-ordinate, w, can also be defined but is redundant.

$$u = \frac{2x}{(6y-x+1.5)} \tag{9}$$

$$v = \frac{3y}{(6y-x+1.5)} \tag{10}$$

5.3 CIE YU'V'

This is another linear transformation of Yxy. Y remains unchanged. Difference non-uniformity is further reduced, but still not enough. Again, a third co-ordinate, w', can be defined, but is redundant.

$$u' = \quad u = \quad \frac{2x}{(6y - x + 1.5)} \tag{11}$$

$$v' = \quad 1.5v = \quad \frac{4.5y}{(6y - x + 1.5)} \tag{12}$$

5.4 CIE L*u*v*

This is based on CIE Yu'v' (1976) and is a further attempt to linearise the perceptibility of unit vector colour differences. It is a non-linear colour space, but the conversions are reversible. Colouring information is centered on the colour of the white point of the system, subscript n, (D65 in most TV systems). The non-linear relationship for Y* is intended to mimic the logarithmic response of the eye.

$$L^* = \begin{cases} 116(\frac{Y}{Y_n})^{\frac{1}{3}} - 16 & \text{if } \frac{Y}{Yn} > 0.008856 \\ \\ 903.3(\frac{Y}{Y_n}) & \text{if } \frac{Y}{Yn} \leq 0.008856 \end{cases} \tag{13}$$

$$u^* = 13(L^*)(u' - u'_n) \tag{14}$$

$$v^* = 13(L^*)(v' - v'_n) \tag{15}$$

L* scales from 0 to 100 for relative luminance (Y/Yn) scaling 0 to 1.

There are three other, more meaningful polar parameters which more closely match the human visual experience than do the Cartesian parameters. Chroma, $C*$, Hue, h_{uv} and Psychometric saturation s_{uv}.

$$C^* = \quad (u^{*2} + v^{*2})^{0.5} \tag{16}$$

$$h_{uv} = \quad arctan(\frac{v^*}{u^*}) \tag{17}$$

$$s_{uv} = \quad \frac{C^*}{L^*} \tag{18}$$

Hue is an angle in four quadrants.

5.5 CIE L*a*b*

This is based directly on CIE XYZ (1931) and is another attempt to linearise the perceptibility of unit vector colour differences. Again, it is non-linear, and the conversions are still reversible. Colouring information is referred to the colour of the white point of the system, subscript n. The non-linear relationships for L* a* and b* are the same as for CIELUV and are intended to mimic the logarithmic response of the eye.

$$L^* = \begin{cases} 116(\frac{Y}{Y_n})^{\frac{1}{3}} - 16 & \text{if } \frac{Y}{Yn} > 0.008856 \\ \\ 903.3(\frac{Y}{Y_n}) & \text{if } \frac{Y}{Yn} \leq 0.008856 \end{cases} \tag{19}$$

$$a^* = 500 * (f(X/X_n) - f(Y/Y_n)) \tag{20}$$

$$b^* = 200 * (f(Y/Y_n) - f(Z/Z_n)) \tag{21}$$

$$\text{where} f(t) = \begin{cases} t^{\frac{1}{3}} & \text{if } t > 0.008856 \\ \\ 7.787 * t + 16/116 & \text{if } t \leq 0.008856 \end{cases} \tag{22}$$

Again, L^* scales from 0 to 100.

Again, there are polar parameters that more closely match the visual experience of colours.

$$C^* = (a^{*2} + b^{*2})^{0.5} \tag{23}$$

$$h_{ab} = arctan(\frac{b*}{a*}) \tag{24}$$

Hue is an angle in four quadrants, and there is no saturation term in this system.

When determining CIEL*a*b* or CIEL*u*v* values for CRT displayed colours it is usual to used the CRT's white point as the reference white.

5.6 Colour Difference.

The difference between two measured colours can be expressed using the CIE colour difference formula. To versions are defined, ΔE^*_{uv} for colours in CIEL*u*v* colour space and ΔE^*_{ab} for colours in CIEL*a*b*.

$$\Delta E^*_{uv} = [(\Delta L^*)^2 + (\Delta u^*)^2 + (\Delta v^*)^2]^(0.5) \tag{25}$$

$$\Delta E^*_{ab} = [(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2]^(0.5) \tag{26}$$

For extremely small colour differences there can be problems with the CIE ΔE measures and the CMC colour difference formula may then give better correlation with perceived differences. The CMC colour difference formula is similar to the CIE versions but includes weighting functions for different areas of the colour space. The CMC formula is defined in Hunt's Measuring Colour.

6 Computer Graphics Colour Spaces.

Traditionally the colour spaces used in computer graphics have been designed for specific devices: RGB for CRT displays and CMY(K) for printers. They are typically device dependent (and so meaningless if applied to different devices or the same device under different conditions). Most computer graphics colour spaces are perceptually non–linear, i.e. a perceived colour difference will vary in magnitude throughout the colour space, this makes them inefficient for coding colour information as some areas will enable expression at too high a precision while others at not enough.

Another problem is that many can be unintuitive in use. The specification of a desired colour can be difficult for the novice (try selecting brown using an RGB vector).

A warning. The transforms in this section, particularly between device independent colour spaces or those including CIE parameters, require very accurate information on the characteristic of the display device. Without this information the values obtained may well be misleading.

We assume all values are normalised between 0 and 1, unless otherwise stated.

7 Computer RGB colour space.

This is the colour space produced on a CRT (or similar) display when pixel values are applied to a graphics card. RGB space may be visualised as a cube with the three axis corresponding to red, green and blue. The bottom corner, when red = green = blue = 0 is black, while the opposite top corner, where red = green = blue = 255 (for an 8 bit per channel display system), is white. RGB is frequently used in most computer applications since no transform is required to display information on the screen. For this reason it is commonly the base colour space for most applications.

Conversion of RGB image pixel values to the CIE XYZ tri-stimulus values of the colour displayed on the CRT can be achieved using a two stage process.

Firstly you need to calculated the relationship between input image pixel values and displayed intensity. This relationship is the transfer function, often simplified to gamma. The transfer functions will usually differ for each channel so are best measured independently.

The second stage is to transform between the displayed red, green and blue to the CIE tristimulus values. This is most easily performed using a matrix transform of the following form:

$$\begin{vmatrix} X \\ Y \\ Z \end{vmatrix} = \begin{vmatrix} X_r & X_g & X_b \\ Y_r & Y_g & Y_b \\ Z_r & Z_g & Z_b \end{vmatrix} * \begin{vmatrix} R \\ G \\ B \end{vmatrix} \quad (27)$$

where X, Y, Z are the desired CIE tri-stimulus values, R, G, B are the displayed RGB values obtained from the transfer functions and the 3x3 matrix is the measured CIE tri-stimulus values for your CRT's three channels (i.e. X_r, Y_r, Z_r are the measured CIE tri-stimulus values for the red channel at maximum emission).

If the transfer functions used are relative (i.e. when using a gamma based method on normalised data) this transform will give you relative values. This means that, for example, derived CIE xy chromaticity values will be correct by the absolute luminance given by the CIE Y tristimulus value will not. If you transfer functions are measured absolutely, to give luminance on the CRT, then the tri-stimulus values should be correct (but see below for other problems).

To convert from XYZ to RGB use the inverse form of the matrix given in equation 27:

$$\begin{vmatrix} R \\ G \\ B \end{vmatrix} = \begin{vmatrix} X_r & X_g & X_b \\ Y_r & Y_g & Y_b \\ Z_r & Z_g & Z_b \end{vmatrix}^{(-1)} * \begin{vmatrix} X \\ Y \\ Z \end{vmatrix} \quad (28)$$

Followed by the inverse of the original transfer function.

In many applications in is not possible to measure the tri-stimulus values for your equipment. This requires expensive equipment, either a spectroradiometer or a colorimeter which give CIE values either by cascading a measured spectral power distribution with the CIE colour matching functions (spectroradiometer) or by using special filters to mimic human vision (colorimeter). An alternative method is to use published manufactures data for your display. Using the published CIE xy chromaticity values for the displays three channels and its white point it is possible to calculate suitable transform matrix values.

Firstly calculate the z chromaticity value for each, remembering that $x + y + z = 1$:

Colour	Chromaticity Coordinates
red	$x_r\ y_r\ z_r$
green	$x_g\ y_g\ z_g$
blue	$x_b\ y_b\ z_b$
white	$x_n\ y_n\ z_n$

Then, assuming the relative luminance, Y, is unity:

$$\begin{aligned} a_r x_r + a_g x_g + a_b x_b &= \tfrac{x_n}{y_n} \\ a_r y_r + a_g y_g + a_b y_b &= 1 \\ a_r z_r + a_g z_g + a_b z_b &= \tfrac{z_n}{y_n} \end{aligned} \tag{29}$$

We have three simultaneous equations with three unknowns enabling the constants a_r, a_g, a_b to be found and used in the matrix as follows.

$$\begin{vmatrix} R \\ G \\ B \end{vmatrix} * \begin{vmatrix} a_r x_r & a_g x_g & a_b x_b \\ a_r y_r & a_g y_g & a_b y_b \\ a_r z_r & a_g z_g & a_b z_b \end{vmatrix} = \begin{vmatrix} X \\ Y \\ Z \end{vmatrix} \tag{30}$$

Often, it is not possible to measure the phosphors of your particular display, as this requires either a spectroradiometer or a colorimeter. You can get round this if you have published data for your display's CIE xy chromaticity co-ordinates of each phosphor and the white point. Since the CIE Y tristimulus value always has a value of one you can calculate CIE XYZ for the white point and then (assuming additivity) CIE XYZ for the three phosphors by solving the three simultaneous equations.

[The white point of a display system is the displayed colour that is produced when all three channels are driven to maximum. (e.g. RGB = 255, 255,255). The white point is usually expressed as either CIE xy chromaticity co-ordinates or a correlated colour temperature in Kelvin.]

Now for the problems with this approach. These transforms rely on knowing, accurately, the characteristics of your display system. If this knowledge is inaccurate then so too will the parameters that you derive. Even with accurate CIE tri-stimulus measurements of the display there are still problems, for example many CRT's vary across the faceplate giving slightly different colours at different positions. In addition the colour produced may vary with time (as the system warms up or as the system ages), usually the chromaticity of the three channels varies with luminance level and cross-talk between channels causes distortion. These problems add up which is why measured values are almost always more accurate than those calculated in this manner. Care should therefore be taken when using and quoting parameters obtained in this way.

Warning: The transform from RGB to CIE XYZ only works for display devices. It will not enable you to determine the CIE co-ordinates of, say, a colour patch on a photograph scanned with your favourite scanner. Think about it.

8 CMY(K) (Cyan Magenta Yellow (Black))

CMY(K) is used in printing and for hard copy output. It is very device dependent with the type of inks and paper used as well as the type of printing device. CMY(K) is also unintuitive for a user and again perceptually non–linear with different devices having different transfer characteristics between specified CMY(K) values and the amount of ink laid down on the paper.

Very simple transforms are often quoted to translate RGB colours displayed on screen to CMY(K) values for printing. These are given below and it should be clear why colours printed in this manner rarely match those displayed on screen.

RGB to CMY

$$\begin{aligned} Cyan &= 1 - Red \\ Magenta &= 1 - Green \\ Yellow &= 1 - Blue \end{aligned} \tag{31}$$

CMY to RGB

$$\begin{aligned} Red &= 1 - Cyan \\ Green &= 1 - Magenta \\ Blue &= 1 - Yellow \end{aligned} \tag{32}$$

CMY to CMYK

$$\begin{aligned} Black &= minimum(Cyan, Magenta, Yellow) \\ Cyan &= (Cyan - Black)/(1 - Black) \\ Magenta &= (Magenta - Black)/(1 - Black) \\ Yellow &= (Yellow - Black)/(1 - Black) \end{aligned} \tag{33}$$

CMYK to CMY

$$\begin{aligned} Cyan &= minimum(1, Cyan * (1 - Black) + Black) \\ Magenta &= minimum(1, Magenta * (1 - Black) + Black) \\ Yellow &= minimum(1, Yellow * (1 - Black) + Black) \end{aligned} \tag{34}$$

These cheap and nasty transforms may be fine for printing a bar chart or for spot colour on a newsletter but for even semi-critical applications the colour reproduction is very poor.

A more accurate method is to calculate the displayed CIE tri-stimulus values from you RGB image pixel values (see section 7 and use these as target values for a similar conversion back through the CMY(K) devices chromaticity values and transfer function to the appropriate CMY(K) values. If you want good colour reproduction this is the kind of complexity you will need to go to (you also ought to be reading Charles Poynton's Color and Gamma FAQ's as well as section 12.

9 HSL (Hue Saturation Lightness).

HSL type colour spaces are deformations of an RGB colour cube. If you imagine the RGB cube tipped cube onto the black corner then the line through the cube from black to white defines the lightness axis. The colour is then defined as a position on a circular plane around the lightness axis. Hue is the angle from a nominal point around the circle to the colour while saturation is the radius from the central lightness axis to the colour.

Alternative examples to HSL include HSV (Hue Saturation Value), HSI (Hue Saturation Intensity) and HCI (Hue Chroma Intensity).

The reason for the number of different HSL related colour spaces appears to stem from the mistaken belief that because they describe colour using names similar to those given in section 2.1 they are device independent and correlate well with visual perception. This is not the case, they are simply linear transforms from RGB and as such share all the short comings of RGB. The one and only advantage of these colour spaces is that they allow a user to intuitivly specify colours making them a good choice in user interfaces.

The mix of HSL type colour spaces has produced a series problem, not only do you need to know exactly which version you are dealing with (HSL, HSI etc.) but also the literature contains different transforms for the same colour space. The transforms presented here are therefore intended as illustrations rather than definitive relationships between RGB and all HSL type spaces.

A second problem is that many applications consider the separation of lightness (or its equivalent) to be advantageous. Unfortunately this is usually no more than an approximation of the lightness information in the image, true lightness calculation requires translation into CIE tristimulus values and then to an appropriate colour space (such as CIEL*hs).

9.1 Hue Saturation Value (Travis).

These are the RGB-HSV conversions given by Travis. To convert from RGB to HSV (assuming normalised RGB values) first find the maximum and minimum values from the RGB triplet. Saturation, S, is then:

$$S = \frac{\text{(max - min)}}{\text{max}} \tag{35}$$

and Value, V, is:

$$V = \text{max} \tag{36}$$

The Hue, H, is then calculated as follows. First calculate R'G'B':

$$\begin{aligned} R' &= \frac{\text{max-R}}{\text{max-min}} \\ G' &= \frac{\text{max-G}}{\text{max-min}} \\ B' &= \frac{\text{max-B}}{\text{max-min}} \end{aligned} \tag{37}$$

if saturation, S, is 0 (zero) then hue is undefined (i.e. the colour has no hue therefore it is monochrome) otherwise:

then, if R = max and G = min

$$H = 5 + B' \tag{38}$$

else if R = max and G $\neq$ min

$$H = 1 - G' \tag{39}$$

else if G = max and B = min

$$H = R' + 1 \tag{40}$$

else if G = max and B $\neq$ min

$$H = 3 - B' \tag{41}$$

else if R = max

$$H = 3 + G' \tag{42}$$

otherwise

$$H = 5 - R' \tag{43}$$

Hue, H, is then converted to degrees by multiplying by 60 giving HSV with S and V between 0 and 1 and H between 0 and 360.

To convert back from HSV to RGB first take Hue, H, in the range 0 to 360 and divide by 60:

$$\text{Hex} = \frac{H}{60} \tag{44}$$

Then the values of primary colour, secondary colour, a, b and c are calculated. the primary colour is the integer component of Hex (e.g. in C `floor(Hex) ;`

$$\text{secondary colour} = \text{Hex - primary colour} \tag{45}$$

$$\text{a} = (1 - S)V \tag{46}$$

$$\text{b} = (1 - (S * \text{secondary colour}))V \tag{47}$$

$$\text{c} = (1 - (S * (1 - \text{secondary colour})))V \tag{48}$$

Finally we calculate RGB as follows;

if primary colour = 0 then

$$R = V, G = c, B = a \tag{49}$$

if primary colour = 1 then

$$R = b, G = V, B = a \tag{50}$$

if primary colour = 2 then

$$R = a, G = V, B = c \tag{51}$$

if primary colour = 3 then

$$R = a, G = b, B = V \tag{52}$$

if primary colour = 4 then

$$R = c, G = a, B = V \tag{53}$$

if primary colour = 5 then

$$R = V, G = a, B = b \tag{54}$$

9.2 Hue Saturation and Intensity. (Gonzalez and Woods).

Gonzalez and Woods give a different colour space using intensity rather than value to denote the lightness axis.

RGB to HSI

$$
\begin{aligned}
I &= \frac{R+G+B}{3} && (55)\\
S &= 1-\left(\frac{3}{(R+G+B)}\right)*a && (56)\\
&\text{where a is the minimum of R, G and B} && (57)\\
H &= cos^{-1}\frac{(0.5*(R-G)+(R-B))}{(((R-G)^2+(R-B)(G-B))^{0.5})} && (58)
\end{aligned}
$$

If S = 0, H is meaningless.

If (B/I) $>$ (G/I) then H = 360 - H since H is an angle in degrees we then normalise to 0,1 with H=H/360

HSI to RGB

First we restore H to degrees with H = 360H

If 0 $<$ H $\leq$ 120 then

$$
\begin{aligned}
B &= 1/3(1-S) && (59)\\
R &= 1/3(1+(ScosH)/(cos(60-H)) && (60)\\
G &= 1-(B+R) && (61)
\end{aligned}
$$

If 120 $<$ H $\leq$ 240 then

$$
\begin{aligned}
H &= H-120 && (62)\\
R &= 1/3(1-S) && (63)\\
G &= 1/3(1+(ScosH)/(cos(60-H)) && (64)\\
B &= 1-(R+G) && (65)
\end{aligned}
$$

If 240 $<$ H $\leq$ 360 then

$$
\begin{aligned}
H &= H-240 && (66)\\
G &= 1/3(1-S) && (67)\\
B &= 1/3(1+(ScosH)/(cos(60-H)) && (68)\\
R &= 1-(G+B) && (69)
\end{aligned}
$$

Many alternatives to these transforms exist, Foley et al. for example give alternatives for HSL and HSI. Make sure you know what you've got therefore before you select a transform.

10 TV and allied non-linear systems

All TV standards use non-linear signals, pre-corrected for the non-linear transfer characteristic of the display CRT. It is here that the most confusion exists, and so this is a VERY important section to understand.

A typical CRT has a non-linear voltage-to-light transfer function with a power law usually denoted by gamma. The value of gamma is theoretically 2.5, but is specified as 2.2 in NTSC systems, 2.8 in PAL systems, and is actually nearer to 2.35 for real CRTs. Any signal destined for display on a CRT must be distorted by an inverse law. In practice, that is impossible because a pure power law has infinite slope (gain) at zero (black). TV systems limit the gain near black to a value between 4 and 5 by offsetting the power law. This has the side advantage of increasing saturation in a way that compensates for the display having a dark surround. For example the ITU-BT.709 specification is:-

$$\text{Volts} = \begin{cases} (1+a)\text{Light}^{\text{law}} - a & \text{if Light} > \text{b} \\ \\ \text{slope} * \text{Light} & \text{for Light} \leq \text{b} \end{cases} \tag{70}$$

where a=0.099, law=0.45, b=0.018.

and the gain at zero is 4.5. This law is similar to the formula used for L* (see above).

So for accurate colour calculations, this law (or whichever law was actually applied) must be undone to return to linear signals before doing conversions. The law should be reapplied to the results to get the drive signals for the actual display.

A signal that has been gamma-corrected is shown primed (Y', R', G', B' etc.). In general, undoing the gamma law will return to linear signals, but that is not always true, especially with the Y' signal, which is not directly related to the CIE Y value. It is a shame that the TV industry used Y' for the luminance channel, because it created a great deal of confusion, most of which still exists. But careful reading of the following section shows the way to performing totally accurate colour calculations using any colour system.

10.1 European Y'U'V' (EBU)

European TV (PAL and SECAM coded) uses Y'U'V' components. Y' is similar to perceived luminance, U' and V' carry the colour information and some luminance information and are bipolar (they go negative as well as positive). The symbols U and V here are not related to the U and V of CIE YUV (1960).

This coding is also used in some 525 line systems with PAL subcarriers, particularly in parts of the Americas. The specification here is that of the European Broadcasting Union (EBU). Y' has a bandwidth of 5 MHz in Europe, 5.5 MHz in UK. The U' and V' signals usually have up to 2.5 MHz bandwidth in a component studio system, but can be as little as 600 kHz or less in a VHS recorder. U' and V' always have the same bandwidth as each other. The CRT gamma law is assumed to be 2.8, but camera correction laws are the same as in all other systems (approximately 0.45). The system white point is D65, the chromaticity co-ordinates are:

R:	xr=0.64	yr=0.33
G:	xg=0.29	yg=0.60
B:	xb=0.15	yb=0.06
White:	xn=0.312713	yn=0.329016

The conversion equations for linear signals are:-

$$\begin{aligned} X &= 0.431 * R + 0.342 * G + 0.178 * B \\ Y &= 0.222 * R + 0.707 * G + 0.071 * B \\ Z &= 0.020 * R + 0.130 * G + 0.939 * B \end{aligned} \tag{71}$$

$$\begin{aligned} R &= 3.063 * X - 1.393 * Y - 0.476 * Z \\ G &= -0.969 * X + 1.876 * Y + 0.042 * Z \\ B &= 0.068 * X - 0.229 * Y + 1.069 * Z \end{aligned} \tag{72}$$

the coding equations for non-linear signals are:

$$\begin{aligned} Y' &= 0.299 * R' + 0.587 * G' + 0.114 * B' \\ U' &= 0.493 * (B' - Y') \\ &= -0.147 * R' - 0.289 * G' + 0.436 * B' \\ V' &= 0.877 * (R' - Y') \\ &= 0.615 * R' - 0.515 * G' - 0.100 * B' \end{aligned} \tag{73}$$

$$\begin{aligned} R' &= Y' + 0.000 * U' + 1.140 * V' \\ G' &= Y' - 0.396 * U' - 0.581 * V' \\ B' &= Y' + 2.029 * U' + 0.000 * V' \end{aligned} \tag{74}$$

The conversion equations between linear 709 RGB signals (see later) and EBU RGB signals are:

$$\begin{aligned} Re &= 0.9578 * R7 + 0.0422 * G7 + 0.0000 * B7 \\ Ge &= 0.0000 * R7 + 1.0000 * G7 + 0.0000 * B7 \\ Be &= 0.0000 * R7 + 0.0118 * G7 + 0.9882 * B7 \end{aligned} \tag{75}$$

$$\begin{aligned} R7 &= 1.0440 * Re - 0.0440 * Ge + 0.0000 * Be \\ G7 &= 0.0000 * Re + 1.0000 * Ge + 0.0000 * Be \\ B7 &= 0.0000 * Re - 0.0119 * Ge + 1.0119 * Be \end{aligned} \tag{76}$$

10.2 American Y'I'Q'

American TV (NTSC coded) uses Y'I'Q' components. Again Y' is similar to perceived luminance, I' and Q' carry colour information and some luminance information and are derived by rotating the U'V' vector formed by colour coding as described in section 3.1 by 33 degrees. The Y' signal usually has 4.2 MHz bandwidth in a 525 line system. Originally the I' and Q' signals were to have different bandwidths (0.5 and 1.5 MHz) but they now commonly have the same bandwidth (1 MHz). The coding is also used in some 625 line countries with NTSC subcarriers, again mostly in the Americas. The CRT gamma law is assumed to 2.2. The system white point is Illuminant C, the chromaticity co-ordinates are:-

R:	xr=0.67	yr=0.33
G:	xg=0.21	yg=0.71
B:	xb=0.14	yb=0.08
White:	xn=0.310063	yn=0.316158

The conversion equations for linear signals are:-

$$\begin{aligned} X &= 0.607 * R + 0.174 * G + 0.200 * B \\ Y &= 0.299 * R + 0.587 * G + 0.114 * B \\ Z &= 0.000 * R + 0.066 * G + 1.116 * B \end{aligned} \tag{77}$$

$$\begin{aligned} R &= 1.910 * X - 0.532 * Y - 0.288 * Z \\ G &= -0.985 * X + 1.999 * Y - 0.028 * Z \\ B &= 0.058 * X - 0.118 * Y + 0.898 * Z \end{aligned} \tag{78}$$

The coding equations for non-linear signals are:-

$$\begin{aligned} Y' &= 0.299 * R' + 0.587 * G' + 0.114 * B' \\ I' &= -0.27 * (B' - Y') + 0.74 * (R' - Y') \\ &= 0.596 * R' - 0.274 * G' + 0.322 * B' \\ Q' &= 0.41 * (B' - Y') + 0.48 * (R' - Y') \\ &= 0.212 * R' - 0.523 * G' - 0.311 * B' \end{aligned} \tag{79}$$

$$\begin{aligned} R' &= Y' + 0.956 * I' + 0.621 * Q' \\ G' &= Y' - 0.272 * I' - 0.647 * Q' \\ B' &= Y' - 1.105 * I' + 1.702 * Q' \end{aligned} \tag{80}$$

It is possible to define a transformation matrix between EBU Y'U'V' and NTSC Y'I'Q'. However, this only makes sense if the primaries are the same for the two systems, and clearly they are defined differently. However, over the years, the American NTSC system has changed its primaries several times until they are now very similar to those of the EBU systems. The non-linear connecting equations are:

$$\begin{aligned} I' = & \quad -(\tfrac{0.27}{0.493})U' + (\tfrac{0.74}{0.877})V' \\ = & \quad -0.547667343U' + 0.843785633V' \end{aligned} \tag{81}$$

$$\begin{aligned} Q' = & \quad (\tfrac{0.41}{0.493})U' + (\tfrac{0.48}{0.877})V' \\ & = 0.831643002U' + 0.547320410V' \end{aligned} \tag{82}$$

and:

$$U' = \quad -0.546512701 * I' + 0.842540416 * Q' \tag{83}$$

$$V' = \quad 0.830415704 * I' + 0.546859122 * Q' \tag{84}$$

To all intents and purposes these equations are identical and so one practical set of equations can be used in either direction:

$$\begin{aligned} I' = & \quad -0.547 * U' + 0.843 * V' \\ Q' = & \quad 0.831 * U' + 0.547 * V' \\ U' = & \quad -0.547 * I' + 0.843 * Q' \\ V' = & \quad 0.831 * I' + 0.547 * Q' \end{aligned} \tag{85}$$

The conversion equations relating NTSC RGB signals to EBU and 709 are:

$$\begin{aligned} R_{ntsc} = & \quad 0.6984 * R_{ebu} + 0.2388 * G_{ebu} + 0.0319 * B_{ebu} \\ G_{ntsc} = & \quad 0.0193 * R_{ebu} + 1.0727 * G_{ebu} - 0.0596 * B_{ebu} \\ B_{ntsc} = & \quad 0.0169 * R_{ebu} + 0.0525 * G_{ebu} + 0.8450 * B_{ebu} \end{aligned} \tag{86}$$

$$\begin{aligned} R_{ebu} = & \quad 1.4425 * R_{ntsc} - 0.3173 * G_{ntsc} - 0.0769 * B_{ntsc} \\ G_{ebu} = & \quad -0.0275 * R_{ntsc} + 0.9350 * G_{ntsc} + 0.0670 * B_{ntsc} \\ B_{ebu} = & \quad -0.0272 * R_{ntsc} - 0.0518 * G_{ntsc} + 1.1081 * B_{ntsc} \end{aligned} \tag{87}$$

and:

$$\begin{aligned} R_{ntsc} = & \quad 0.6698 * R_{709} + 0.2678 * G_{709} + 0.0323 * B_{709} \\ G_{ntsc} = & \quad 0.0185 * R_{709} + 1.0742 * G_{709} - 0.0603 * B_{709} \\ B_{ntsc} = & \quad 0.0162 * R_{709} + 0.0432 * G_{709} + 0.8551 * B_{709} \end{aligned} \tag{88}$$

$$\begin{aligned} R_{709} = & \quad 1.5073 * R_{ntsc} - 0.3725 * G_{ntsc} - 0.0832 * B_{ntsc} \\ G_{709} = & \quad -0.0275 * R_{ntsc} + 0.9350 * G_{ntsc} + 0.0670 * B_{ntsc} \\ B_{709} = & \quad -0.0272 * R_{ntsc} - 0.0401 * G_{ntsc} + 1.1677 * B_{ntsc} \end{aligned} \tag{89}$$

10.3 SMPTE-C RGB

SMPTE-C is the current colour standard for broadcasting in America, the old NTSC standard for primaries is no longer in wide use because the primaries of the system have gradually shifted towards those of the EBU (see section 6.2). In all other respects, SMPTE-C is the same as NTSC. The CRT gamma law is assumed to be 2.2. The white point is now D65, and the chromaticities are:

R:	xr=0.630	yr=0.340
G:	xg=0.310	yg=0.595
B:	xb=0.155	yb=0.070
White:	xn=0.312713	yn=0.329016

The conversion equations for linear signals are:

$$\begin{aligned} X &= 0.3935 * R + 0.3653 * G + 0.1916 * B \\ Y &= 0.2124 * R + 0.7011 * G + 0.0866 * B \\ Z &= 0.0187 * R + 0.1119 * G + 0.9582 * B \end{aligned} \tag{90}$$

$$\begin{aligned} R &= 3.5058 * X - 1.7397 * Y - 0.5440 * Z \\ G &= -1.0690 * X + 1.9778 * Y + 0.0352 * Z \\ B &= 0.0563 * X - 0.1970 * Y + 1.0501 * Z \end{aligned} \tag{91}$$

The coding equations for non-linear signals are the same as for NTSC:

$$\begin{aligned} Y' &= 0.299 * R' + 0.587 * G' + 0.114 * B' \\ I' &= -0.27 * (B' - Y') + 0.74 * (R' - Y') \\ &= 0.596 * R' - 0.274 * G' + 0.322 * B' \\ Q' &= 0.41 * (B' - Y') + 0.48 * (R' - Y') \\ &= 0.212 * R' - 0.523 * G' - 0.311 * B' \end{aligned} \tag{92}$$

$$\begin{aligned} R' &= Y' + 0.956 * I' + 0.621 * Q' \\ G' &= Y' - 0.272 * I' - 0.647 * Q' \\ B' &= Y' - 1.105 * I' + 1.702 * Q' \end{aligned} \tag{93}$$

and the same conversion equations work between EBU and SMPTE-C components:

$$\begin{aligned} I' &= -0.547 * U' + 0.843 * V' \\ Q' &= 0.831 * U' + 0.547 * V' \\ U' &= -0.547 * I' + 0.843 * Q' \\ V' &= 0.831 * I' + 0.547 * Q' \end{aligned} \tag{94}$$

The conversion equations relating SMPTE-C RGB signals to EBU and 709 signals are:

$$\begin{aligned} R_{smptec} &= \quad 1.1123 * R_{ebu} - 0.1024 * G_{ebu} - 0.0099 * B_{ebu} \\ G_{smptec} &= \quad -0.0205 * R_{ebu} + 1.0370 * G_{ebu} - 0.0165 * B_{ebu} \\ B_{smptec} &= \quad 0.0017 * R_{ebu} + 0.0161 * G_{ebu} + 0.9822 * B_{ebu} \end{aligned} \tag{95}$$

$$\tag{96}$$

$$\begin{aligned} R_{ebu} &= \quad 0.9007 * R_{smptec} + 0.0888 * G_{smptec} + 0.0105 * B_{smptec} \\ G_{ebu} &= \quad 0.0178 * R_{smptec} + 0.9658 * G_{smptec} + 0.0164 * B_{smptec} \\ B_{ebu} &= \quad -0.0019 * R_{smptec} - 0.0160 * G_{smptec} + 1.0178 * B_{smptec} \end{aligned} \tag{97}$$

and:

$$\begin{aligned} R_{smptec} &= \quad 1.0654 * R_{709} - 0.0554 * G_{709} - 0.0010 * B_{709} \\ G_{smptec} &= \quad -0.0196 * R_{709} + 1.0364 * G_{709} - 0.0167 * B_{709} \\ B_{smptec} &= \quad 0.0016 * R_{709} + 0.0044 * G_{709} + 0.9940 * B_{709} \end{aligned} \tag{98}$$

$$\begin{aligned} R_{709} &= \quad 0.9395 * R_{smptec} + 0.0502 * G_{smptec} + 0.0103 * B_{smptec} \\ G_{709} &= \quad 0.0178 * R_{smptec} + 0.9658 * G_{smptec} + 0.0164 * B_{smptec} \\ B_{709} &= \quad -0.0016 * R_{smptec} - 0.0044 * G_{smptec} + 1.0060 * B_{smptec} \end{aligned} \tag{99}$$

10.4 ITU.BT-601 Y'CbCr

This is the international standard for digital coding of TV pictures at 525 and 625 line rates. It is independent of the scanning standard and the system primaries, therefore there are no chromaticity co-ordinates, no CIE XYZ matrices, and no assumptions about white point or CRT gamma. It deals only with the digital representation of R'G'B' signals in Y'CbCr form. The non-linear coding matrices are:

$$\begin{aligned} Y' &= \quad 0.299 * R' + 0.587 * G' + 0.114 * B' \\ Cb &= \quad -0.169 * R' - 0.331 * G' + 0.500 * B' \\ Cr &= \quad 0.500 * R' - 0.419 * G' - 0.081 * B' \end{aligned} \tag{100}$$

$$\begin{aligned} R' &= \quad Y' + 0.000 * U' + 1.403 * V' \\ G' &= \quad Y' - 0.344 * U' - 0.714 * V' \\ B' &= \quad Y' + 1.773 * U' + 0.000 * V' \end{aligned} \tag{101}$$

10.5 ITU.BT-709 HDTV studio production in Y'CbCr

This is a recent standard, defined only as an interim standard for HDTV studio production. It was defined by the CCIR (now the ITU) in 1988, but is not yet recommended for use in broadcasting. The primaries are the R and B from the EBU, and a G which is midway between SMPTE-C and EBU. The CRT gamma law is assumed to be 2.2. White is D65. The chromaticities are:

R:	xr=0.64	yr=0.33
G:	xg=0.30	yg=0.60
B:	xb=0.15	yb=0.06
White:	xn=0.312713	yn=0.329016

The conversion equations for linear signals are:

$$\begin{aligned} X &= 0.412 * R + 0.358 * G + 0.180 * B \\ Y &= 0.213 * R + 0.715 * G + 0.072 * B \\ Z &= 0.019 * R + 0.119 * G + 0.950 * B \end{aligned} \tag{102}$$

$$\begin{aligned} R &= 3.241 * X - 1.537 * Y - 0.499 * Z \\ G &= -0.969 * X + 1.876 * Y + 0.042 * Z \\ B &= 0.056 * X - 0.204 * Y + 1.057 * Z \end{aligned} \tag{103}$$

The coding equations for non-linear signals are:

$$\begin{aligned} Y' &= 0.2215 * R' + 0.7154 * G' + 0.0721 * B' \\ Cb &= -0.1145 * R' - 0.3855 * G' + 0.5000 * B' \\ Cr &= 0.5016 * R' - 0.4556 * G' - 0.0459 * B' \end{aligned} \tag{104}$$

$$\begin{aligned} R' &= Y' + 0.0000 * Cb + 1.5701 * Cr \\ G' &= Y' - 0.1870 * Cb - 0.4664 * Cr \\ B' &= Y' - 1.8556 * Cb + 0.0000 * Cr \end{aligned} \tag{105}$$

The conversion equations between linear 709 RGB signals and EBU RGB signals are:

$$\begin{aligned} Re &= 0.9578 * R7 + 0.0422 * G7 + 0.0000 * B7 \\ Ge &= 0.0000 * R7 + 1.0000 * G7 + 0.0000 * B7 \\ Be &= 0.0000 * R7 + 0.0118 * G7 + 0.9882 * B7 \end{aligned} \tag{106}$$

$$\begin{aligned} R7 &= 1.0440 * Re - 0.0440 * Ge + 0.0000 * Be \\ G7 &= 0.0000 * Re + 1.0000 * G7 + 0.0000 * Be \\ B7 &= 0.0000 * Re - 0.0119 * Ge + 1.0119 * Be \end{aligned} \tag{107}$$

10.6 SMPTE-240M Y’PbPr

This one of the developments of NTSC component coding, in which the B primary and white point were changed. The CRT gamma law is assumed to be 2.2. The white point is D65, chromaticity co-ordinates are:

R:	xr=0.67	yr=0.33
G:	xg=0.21	yg=0.71
B:	xb=0.15	yb=0.06
White:	xn=0.312713	yn=0.329016

The conversion equations for linear signals are:

$$\begin{aligned} X = & \ 0.567 * R + 0.190 * G + 0.193 * B \\ Y = & \ 0.279 * R + 0.643 * G + 0.077 * B \\ Z = & \ 0.000 * R + 0.073 * G + 1.016 * B \end{aligned} \tag{108}$$

$$\begin{aligned} R = & \ 2.042 * X - 0.565 * Y - 0.345 * Z \\ G = & \ -0.894 * X + 1.815 * Y + 0.032 * Z \\ B = & \ 0.064 * X - 0.129 * Y + 0.912 * Z \end{aligned} \tag{109}$$

The coding equations for non-linear signals are:

$$\begin{aligned} Y' = & \ 0.2122 * R' + 0.7013 * G' + 0.0865 * B' \\ Pb = & \ -0.1162 * R' - 0.3838 * G' + 0.5000 * B' \\ Pr = & \ 0.5000 * R' - 0.4451 * G' - 0.0549 * B' \end{aligned} \tag{110}$$

$$\begin{aligned} R' = & \ Y + 0.0000 * P_b + 1.5756 * P_r \\ G' = & \ Y - 0.2253 * P_b + 0.5000 * P_r \\ B' = & \ Y + 1.8270 * P_b + 0.0000 * P_r \end{aligned} \tag{111}$$

The conversion equations relating SMPTE 240 RGB to EBU and 709 primaries are:

$$\begin{aligned} R_{240} & \ 0.7466 * R_e + 0.2534 * G_e + 0.0000 * B_e \\ G_{240} = & \ 0.0187 * R_e + 0..9813 * G_e + 0.0000 * B_e \\ B_{240} = & \ 0.0185 * R_e + 0.0575 * G_e + 0.9240 * B_e \end{aligned} \tag{112}$$

$$\begin{aligned} R_e = & \ 1.3481 * R_{240} - 0.3481 * G_{240}0 + 0.0000 * B_{240} \\ G_e = & \ -0.0257 * R_{240} + 1.0257 * G_{240} + 0.0000 * B_{240} \\ B_e = & \ -0.0254 * R_{240} - 0.0568 * G_{240} + 1.0822 * B_{240} \end{aligned} \tag{113}$$

and:

$$\begin{aligned} R_{240} = & \ 0.7151 * R_{709} + 0.2849 * G_{709} + 0.0000 * B_{709} \\ G_{240}0 = & \ 0.0179 * R_{709} + 0.9821 * G_{709} + 0.0000 * B_{709} \\ B_{240} = & \ 0.0177 * R_{709} + 0.0472 * G_{709} + 0.9350 * B_{709} \end{aligned} \tag{114}$$

$$
\begin{aligned}
R_{709} &= \ \ 1.4086 * R_{240} - 0.4086 * G_{240} + 0.0000 * B_{240} \\
G_{709} &= -0.0257 * R_{240} + 1.0457 * G_{240} + 0.0000 * B_{240} \\
B_{709} &= -0.0254 * R_{240} - 0.0440 * G_{240} + 1.0695 * B_{240}
\end{aligned} \tag{115}
$$

11 Kodak PhotoYCC Colour Space.

The Kodak PhotoYCC colour space was designed for encoding images with the PhotoCD system. It is based on both CCIR Recommendations 709 and 601-1, having a colour gamut defined by the CCIR 709 primaries and a luminance - chrominance representation of colour like CCIR 601-1's YCbCr.

Since PhotoYCC encoded images are designed for display on both television / video and on computer graphics systems PhotoYCC itself is a bit of a mix between television and computer graphics standards for encoding colour.

To encode data, a transfer function (gamma correction) is first applied;

For R,G,B $>$ 0.018

$$
\begin{aligned}
R' &= 1.099 * R^0.45 - 0.099 \\
G' &= 1.099 * G^0.45 - 0.099 \\
B' &= 1.099 * B^0.45 - 0.099
\end{aligned} \tag{116}
$$

For R,G,B, $\leq$ 0.018

$$
\begin{aligned}
R' &= 4.5 * R \\
G' &= 4.5 * G \\
B' &= 4.5 * B
\end{aligned} \tag{117}
$$

Secondly the R'G'B' data is transformed into PhotoYCC data,

$$
\begin{aligned}
\text{Luma} &= 0.299R' + 0.587G' + 0.114B' \\
\text{Chroma1} &= -0.299R' - 0.587G' + 0.886B' \\
\text{Chroma2} &= 0.701R' - 0.587G' - 0.114B'
\end{aligned} \tag{118}
$$

Finally the floating point values are stored as 8 bit integers;

$$
\begin{aligned}
\text{Luma 8bit} &= (255/1.402)\text{Luma} \\
\text{Chroma1 8bit} &= 111.40 * \text{Chroma1} + 156 \\
\text{Chroma2 8bit} &= 135.64 * \text{Chroma2} + 137
\end{aligned} \tag{119}
$$

The unbalanced scale difference between Chroma1 and Chroma2 is designed, according to Kodak, to follow the typical distribution of colours in real scenes.

Kodak YCC can store more information than current display devices can cope with (it allows negative RGB values), the transforms from YCC to RGB are therefore not simply the inverse of RGB to YCC, they depend on the target display system.

Converting PhotoYCC data to RGB 24bit data for display by computers on CRT's is achieved as follows;

Firstly normal Luma and Chroma data are recovered;

$$
\begin{aligned}
\text{Luma} &= 1.3584 * \text{Luma 8bit} \\
\text{Chroma1} &= 2.2179 * (\text{Chroma1 8bit} - 156) \\
\text{Chroma2} &= 1.8215 * (\text{Chroma2 8bit} - 137)
\end{aligned} \tag{120}
$$

For display primaries that are, or are very close to, CCIR Recommendation 709 primaries in their chromaticities, then

$$
\begin{aligned}
R' &= L + \text{Chroma2} \\
G' &= L - 0.194\text{Chroma1} - 0.509\text{Chroma2} \\
B' &= L + \text{Chroma1}
\end{aligned} \tag{121}
$$

Two things to watch are;

a) this results in RGB values from 0 to 346 (instead of the more usual 0 to 255) a look-up-table is usually used to convert these through a non-linear function to 8 bit data. For example;

$$
\begin{aligned}
Y &= (255/1.402) * Y' \\
C1 &= 111.40 * C1' + 156 \\
C2 &= 135.64 * C2' + 137
\end{aligned} \tag{122}
$$

b) if the display phosphors differ from CCIR 709 primaries then further conversion will be necessary, possibly through an intermediate device independent colour space such as CIE XYZ.

Converting stored PhotoYCC data to RGB signal voltages for display on television's is achieved as follows;

Firstly normal Luma and Chroma data are recovered;

$$
\begin{aligned}
\text{Luma} &= 1.3584 * \text{Luma 8bit} \\
\text{Chroma1} &= 2.2179 * (\text{Chroma1 8bit} - 156) \\
\text{Chroma2} &= 1.8215 * (\text{Chroma2 8bit} - 137)
\end{aligned} \tag{123}
$$

the RGB display voltages are then as follows;

$$
\begin{aligned}
R' &= (L + \text{Chroma2})/353.2 \\
G' &= (L - 0.194\text{Chroma1} - 0.509\text{Chroma2})/353.2 \\
B' &= (L + \text{Chroma1})/353.2
\end{aligned} \tag{124}
$$

Note, the gamma corrected RGB values (R'G'B') are fine so long as you haven't done anything silly - like calibrate you CRT for linear luminance ;-)

12 Colour appearence.

The CIE system allows the measurement of colour according to characteristics of human vision. A CIE specification will enable a colour to be made to match another and can be used to predict visual differences between colours. What the CIE system does not tell us is the appearance of a colour. This is because the appearance of colour is influenced by many factors, including the type of lighting, the geometry of the colour surface, and the characteristics of surrounding colours that are in the visual field. To illustrate this effect try placing a small piece of grey card on a variety of different coloured backgrounds, notice how the appearance of the grey patch changes. You can also experiment with this effect using a simple paint program. Draw a number of different coloured boxes on screen and place a small mid-grey coloured box in the centre of each, notice how the appearance of the grey boxes differ even though they are the same colour and, if measured, would give identical CIE values.

This difference between colour measurement and colour appearance can result in problems when trying to match colours between different devices, for example between a hard copy output and an image on a soft display. The CRT will have a specific white point and elements on screen around the image as well as the screen bezel and surroundings will affect the appearance of the image. When printed the printing stock, type of lighting and surrounding elements on the page will have a different affect on appearance. These differences result in colour reproduction that does not appear to match when assessed visually but does match when measured colorimetrically.

It is possible to solve this problem by using a colour appearance space. As yet no international standards exist in this area but several workers have presented methods for deriving and working in colour appearance spaces. One example is Hunt's appearance space which uses colorimetric measurements of the image as well as of the illuminant, reference white and various regions of the visual field to produce parameters that correlate with the CIE's definitions of perceived colour – namely lightness, brightness, hue, chroma, colourfulness and saturation.

The advantage of such an appearance space is that it enables us to predict what a colour will look like when viewed by a (typical) observer in a variety of conditions. Using such a system it is possible to get accurate colour reproduction between soft display and hard copy at the expense of computational complexity.

13 The Colour Reproduction Index.

It is often desirable to have a measure of the success (or lack of) of a system in reproducing colour. The use of CIE ΔE^* colour differences between an original and sample image can be used but a large number of differences need to be evaluated in order to have an assessment of the system's performance in different regions of the system's colour gamut. Another problem is that ΔE^* measures take no account of the changes in viewing conditions which may affect colour appearance. An alternative is to use the Colour Reproduction Index (CRI) devised by Mike Pointer at Kodak. The CRI takes a selection of colour's scattered throughout the systems gamut (typically those on a MacBeth ColorChecker Chart) and calculates correlates of perceived colour appearance using Hunt's model of colour vision. These values are weighed according to the visual importance of different colours and compared to the original test image to produce a single value, the CRI, which describes the systems capability.

14 Some references and bedtime reading.

Any collection of recommended reading of this subject is likely to be incomplete, so here are a few to get you going.

14.1 On–line references.

The CIE's homepage:
`http://www.cie.co.at/cie/home.html`
Charles Poynton's Color and Gamma FAQ page:
`http://www.poynton.com/Poynton-color.html`
University of Derby, Colour Research Group's colour links:
`http://ziggy.derby.ac.uk/web/colour.html`
This document:
`http://www.poynton.com/PDFs/coloureq.pdf`

And also on USENET try:
sci.engr.color
comp.graphics hierarchy
sci.image.processing

14.2 Real Paper References.

14.2.1 General Colour Theory.

"The Reproduction of Colour in Photography, Printing and Television", R.W.G. Hunt, Fountain Press, Tolworth, England. [was out of print, new fifth edition available]

"Measuring Colour.", second edition, R.W.G. Hunt, Ellis Horwood 1991.

"Colour science — Concepts and methods, quantitive data and formalae.", Gunter, Wyszecki and Stiles, WS Wiley and Sons INC NY 1982

"CIE Colorimetry.", Official recommendations of the International Commission on Illumination, Publication 15.2 1986

14.2.2 RGB to CIE conversion.

"Calibration of a computer controlled color monitor", Brainard, D.H, Color Research & Application, 14, 1, pp 23-34 (1989).

"An inexpensive scheme for calibration of a colour monitor in terms of CIE standard coordinates", W.B. Cowan, Computer Graphics, Vol. 17 No. 3, 1983.

"CRT Colorimetry: Part 1 Theory and Practice, Part 2 Metrology", Berns, R.S., Motta, R.J. and Gorzynski, M.E., Color Research and Application, 18, (1993).

14.2.3 Colour in TV and Computer Graphics.

"Colour Science in Television and Display Systems.", W.N. Sproson, Adam Hilger Ltd, 1983. ISBN 0-85274-413-7

"Effective Color Displays. Theory and Practice.", Travis, D, Academic Press, 1991. ISBN 0-12-697690-2 (This contains C source code for many colour space conversions.)

"Computer Generated Colour.", R. Jackson, L. MacDonald, K. Freeman, John Wiley and Sons, 1994.

14.2.4 Colour and printing.

Color and Its Reproduction.", G.G. Field, Graphics Arts Technical Foundation, 1988, pp. 320-9

14.2.5 Gamma and transfer functions.

"Gamma and its disguises: The nonlinear mappings of intensity in perception, CRT's, Film and Video.", C. A. Poynton, SMPTE Journal, December 1993

14.2.6 Colour in Digital Image Processing and Computer Graphics.

"Digital Image Processing.", Rafael C. Gonzalez and Richard E. Woods, Addision Wesley, 1992. (Chapter 4.6)

"Fundamentals of Digital Image Processing.", Anil K. Jain, Prentice-Hall International, 1989. (Chapter 3)

"Computer graphics : principles and practices.", James D. Foley, et al. 2nd ed. Addison-Wesley, c1990.

14.2.7 Other Colour Related.

"On the Gun Independence and Phosphor Consistency of Color Video Monitors.", W.B. Cowan N. Rowell, Color Research and Application, V.11 Supplement 1986

"Precision requirements for digital color reproduction.", M Stokes, MD Fairchild and RS Berns, ACM Transactions on graphics, v11 n4 1992

"The colorimetry of self luminous displays - a bibliography.", CIE Publication n.87, Central Bureau of the CIE, Vienna 1990.

"Fully Utilizing Photo CD Images, Article No. 4, PhotoYCC Colour Encoding and Compression Schemes.", Eastman Kodak Company, Rochester NY (USA) (1993).

15 Footnotes & Disclaimer.

A big thank you to all who have contributed to this document. Please feel free to send comments, suggestions, and error reports to Charles Poynton (poynton@poynton.com).

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